$X^2$ test for Independence

3x3 Table

Ex: Does "test failure" reduce academic aspirations and thereby contribute to the decision to drop out of school? A survey of 283 students randomly selected from schools with low graduation rates.

The contingency table below reports results to the question "Do tests required for graduation discourage students from staying in school"? Does there appear to be a relation
between the school's location and the students' responses.
(Use $\alpha = .05$)

<table>
<thead>
<tr>
<th>School Location</th>
<th>Urban</th>
<th>Suburb</th>
<th>Rural</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>57(57.86)</td>
<td>27(31.48)</td>
<td>47(41.66)</td>
</tr>
<tr>
<td>No</td>
<td>23(22.53)</td>
<td>16(12.25)</td>
<td>12(16.22)</td>
</tr>
<tr>
<td>Unsure</td>
<td>45(44.61)</td>
<td>25(24.27)</td>
<td>31(32.12)</td>
</tr>
<tr>
<td>Totals</td>
<td>125</td>
<td>68</td>
<td>90</td>
</tr>
</tbody>
</table>

$H_0$: School location and student response are indep.
$H_1$: School location and student response are dependent

$\alpha = 0.05$

For $r \times c$ tables, the test statistic is not the same as for $2 \times 2$ tables.

Test statistic (for $r \times c$ tables):

$$\chi^2* = \sum \frac{(O - E)^2}{E}$$

where $O$ represents the observed cell frequencies; and
the expected cell frequencies \( E \) are computed as usual

\[
E = \frac{\text{row total} \times \text{col total}}{n}
\]

For our example

\[
\chi^2* = \frac{(57 - 57.86)^2}{57.86} + \frac{(27 - 31.48)^2}{31.48}
\]

\[
+ \ldots + \frac{(31 - 32.12)^2}{32.12}
\]

\[
= .013 + .638 + .684 + .010 + 1.148
\]

\[
+ 1.098 + .003 + .022 + .039
\]

\[
= 3.655
\]
Critical Value: Table C

df = (r-1)(c-1)
    = (3-1)(3-1) = 4

α = .05

χ² = 9.49

Reject Ho if χ²* > χ²

Since 3.655 < 9.49

do not reject Ho.

We conclude that the evidence is not sufficient to show a significant association between school location and response.
Assumptions:
Some as for 2x2 table