Probability and Compound Events

Def: An event comprised of two (or more) events is a compound event.

Ex: Roll fair 6-sided die once

\[ E_1 = \text{roll even number} = \{ 2, 4, 6 \} \]

\[ E_2 = \text{roll number beginning with "t"} = \{ 2, 3 \} \]
The compound event

\[ E_1 \cup E_2 = E_1 \text{ or } E_2 \text{ (or both)} \]

occurs if \( E_1 \) happens or
if \( E_2 \) happens or
if both events happen

So

\[ E_1 \cup E_2 = \text{ roll an even number or a number beginning with "t" or both} \]

\[ = \{2, 3, 4, 6\} \]

Thus

\[ P(E_1 \cup E_2) = P(E_1 \text{ or } E_2 \text{ or both}) = \frac{4}{6} \]
The Addition Law
(The "OR" LAW)

\[ P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2) \]

Ex: Roll fair die once
\[ E_1 = \text{roll even} \]
\[ E_2 = \text{roll number starting with "t"} \]
\[ E_1 \cap E_2 = \text{roll even starting with "t"} \]
\[ E_1 \text{ and } E_2 = \{2\} \]
So
\[ P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) \]
\[ = \frac{3}{6} + \frac{6}{6} - \frac{1}{6} \]
\[ = \frac{4}{6} \]
Mutually Exclusive Events

Def: Event $E_1$ and event $E_2$ are mutually exclusive if the occurrence of one event precludes the occurrence of the other event.

Note: Two events are mutually exclusive if they cannot both occur simultaneously.

In symbols

$$P(E_1 \text{ and } E_2) = 0$$

EX: Roll a fair die once

$E_1 =$ Roll even

$E_2 =$ Roll a 5

These events are mutually exclusive.
SPECIAL ADDITION LAW

IF event $E_1$ and event $E_2$ are mut. excl. then

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

EX: (Continued)

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$
$$= \frac{3}{6} + \frac{1}{6}$$
$$= \frac{4}{6}$$
Note: If \( E_1, E_2 \) are mutually exclusive, then

\[
P(E_1 | E_2) = 0
\]

and

\[
P(E_2 | E_1) = 0
\]

and

\[
P(E_1 \cap \neg E_2) = 0
\]
The compound event
\[ E_1 \cap E_2 = E_1 \text{ and } E_2 \]
occurs if and only if both event \( E_1 \) occurs and event \( E_2 \) occurs.

So

\[ E_1 \cap E_2 = \text{ roll a number which is even and begins with } "t" \]

\[ = \{2\} \]

Thus

\[ P(E_1 \cap E_2) = \frac{1}{6} \]
Probability Trees

Ex: Select two marbles w/o replacement from box containing 3 Red and 2 White

First Marble

Second Marble

\[ P(R \text{ and } R) = 0.6 \times 0.5 = 0.30 \]

\[ P(R \text{ and } W) = 0.6 \times 0.5 = 0.30 \]

\[ P(W \text{ and } R) = 0.4 \times 0.75 = 0.30 \]

\[ P(W \text{ and } W) = 0.4 \times 0.25 = 0.10 \]