Poisson Dist.

Poisson dist. is concerned with "the number of random events per unit time, or space, or both."

Note: Here "random" implies that there is a constant prob. that the event will occur in one unit of time or space.
Poisson Random Variables

Examples:

1) The number of people arriving at a check-out lane in Giant Eagle in a 5 minute interval.

2) The number of earthworms in one cubic yard of dirt.
3) The number of grass sprouts appearing in one week of a seeded one square yard plot of land.
General Poisson Distribution

The Poisson dist. is an appropriate model for experiments in which:

1) random variable $X$ counts the number of occurrences of some event of interest in a unit of time or space (or both)

2) the events occur randomly

3) the mean number of events per unit of time/space is constant

4) random variable $X$ has no fixed upper limit

Then the random variable $X$ possesses a Poisson dist.
and we can use the general Poisson dist. formula to compute probs.

Suppose $\lambda$ is the mean number of events per unit time or space, and $X$ denotes the number of possible events per unit time or space. Then

$$P(X) = \frac{e^{-\lambda} \lambda^x}{x!}$$

for $x=0,1,2,\ldots$
Note:

\[ e \approx 2.7182818 \ldots \]

\[ e^x \]
Calculating Poisson Probs.

Ex: Suppose arrivals at a check-out line "average" 2 persons each 5 minutes.

a) Find the prob. that no people arrive at the check-out line in the next 5 min.

Unit of time = 5 min.

\[ \lambda = 2 \]

\[ X = 0 \]

So \[ P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \]

for \[ x = 0, 1, 2, \ldots \]
So \( P(0) = \frac{e^{-2} 2^0}{0!} \)

\[ = \frac{(.1353)(1)}{1} \]

\[ = 0.1353 \]

b) Find the probability that 3 people arrive at the check-out line in the next 5 min.

\[ P(3) = \frac{e^{-2} 2^3}{3!} \]

\[ = \frac{(.1353)(8)}{6} \]

\[ = \frac{1.804}{6} \]

\[ = .1804 \]
c) Find the prob. that 4 people arrive at check-out in the next 15 min.

\[ \text{time unit} = 15 \text{ min.} \]

\[ \frac{2 \text{ persons}}{5 \text{ min}} = \frac{6 \text{ people}}{15 \text{ min}}. \]

So \( \lambda = 6 \)

and \( X = 4 \)

\[ P(4) = e^{-\lambda} \frac{\lambda^4}{4!} \]

\[ = e^{-6} \frac{6^4}{4!} \]

\[ = (0.00247875)(1296) \frac{24}{24} \]

\[ = 0.13385 \]
Mean, Std. Dev. of Poisson Dist.

\[ \text{mean} = \mu = \lambda \]

\[ \text{std. dev.} = \sigma = \sqrt{\lambda} \]

Ex: \[ \lambda = 5 = \mu \]

\[ \sigma = \sqrt{5} = 2.236 \]