Chapter 8 Solutions

8.1. With $\hat{p} = 0.89$ and $n = 1200$, the standard error is $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/n} \doteq 0.009032$, so the margin of error for 90% confidence is $1.645\,\text{SE}_{\hat{p}} \doteq 0.01486$.

8.2. $\hat{p} = \frac{336}{1200} = 0.28$, and $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/1200} \doteq 0.01296$, so the margin of error for 95% confidence is $1.96\,\text{SE}_{\hat{p}} \doteq 0.02540$ and the interval is 0.2546 to 0.3054.

8.3. **(a)** H_0: $p = 0.72$ vs. H_a: $p \neq 0.72$, where p is the proportion of working students at your university. **(b)** With $\hat{p} = 0.77$, the standard error is $\sigma_{\hat{p}} = \sqrt{(0.72)(0.28)/100} \doteq 0.04490$ and the test statistic is $z = \frac{0.77-0.72}{0.04490} \doteq 1.11$. The P-value is 0.2670. **(c)** No, this result gives no reason to believe that the proportion is different from 0.72.

8.4. **(a)** H_0: $p = 0.83$ vs. H_a: $p > 0.83$, where p is the proportion of undergraduates who own a cell phone. **(b)** With $\hat{p} = 0.89$, the standard error is $\sigma_{\hat{p}} = \sqrt{(0.89)(0.11)/1200} \doteq 0.01084$ and the test statistic is $z = \frac{0.89-0.83}{0.01084} \doteq 5.53$. A test statistic as large as this clearly significant ($P < 0.00005$). **(c)** We have strong evidence that cell phone ownership has increased; the evidence is significant at $\alpha = 0.05$ (or even a much smaller significance level).

8.5. We can achieve that margin of error with 90% confidence with a smaller sample. With $p^* = 0.5$ (as in Example 8.6), we compute $n = \left(\frac{1.645}{(2)(0.03)}\right)^2 \doteq 751.67$, so we need a sample of 752 students.

8.6. Use $p^* = 0.77$ (the estimate from our sample of 100 students). We compute $n = \left(\frac{1.96}{0.02}\right)^2 (0.77)(0.23) \doteq 1700.9$, so we need a sample of 1701 students.

8.7. Recall the text's guidelines: Large-sample intervals can be used when the number of successes and the number of failures are both at least 15. **(a)** Yes: With 30 successes and 20 failures, a large-sample interval can be used. **(b)** Yes: With 15 successes and 75 failures, a large-sample interval can be used. **(c)** No: There are only 2 successes and 8 failures. **(d)** No: There are only 10 failures. **(e)** No: There are only 10 failures.

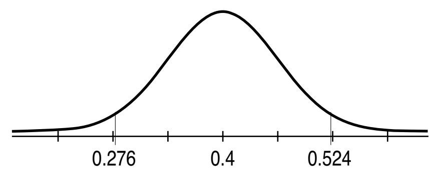
Note: *The plus four confidence interval procedure can be used whenever $n \geq 10$, so it could be used in all of these circumstances.*

8.8. Recall the text's guidelines: Large-sample intervals can be used when the number of successes and the number of failures are both at least 15. **(a)** No: There are only 4 successes and 4 failures. **(b)** No: There are only 13 successes. **(c)** No: There are only 12 failures. **(d)** No: There are no failures. **(e)** Yes: With 22 successes and 28 failures, a large-sample interval can be used.

Note: *The plus four confidence interval procedure can be used whenever $n \geq 10$, so it could be used for all but (a).*

8.9. **(a)** The margin of error equals z^* times standard error; for 95% confidence, we would have $z^* = 1.96$. **(b)** Use Normal distributions (and a z test statistic) for significance tests involving proportions. **(c)** H_0 should refer to p (the population proportion), not $\hat{p}$ (the sample proportion).

8.10. **(a)** The mean is $\mu = p = 0.4$ and the standard deviation is $\sigma = \sqrt{p(1-p)/n} = \sqrt{0.004} \doteq 0.06325$. **(b)** Normal curve on the right. **(c)** p^* should be either $1.96\sigma \doteq 0.1240$ or $2\sigma \doteq 0.1265$, so the points marked on the curve should be either 0.276 and 0.524 or 0.2735 and 0.5265.



8.11. **(a)** $\hat{p} = \frac{3547}{5594} \doteq 0.6341$. The standard error is $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/5594} \doteq 0.006440$, so the margin of error for 95% confidence is $1.96\,\text{SE}_{\hat{p}} \doteq 0.01262$ and the interval is 0.6214 to 0.6467. This interval was found using a procedure that includes the correct proportion 95% of the time. **(b)** We do not know if those who *did* respond can reliably represent those who did not.

8.12. **(a)** $\hat{p} = \frac{1447}{3469} \doteq 0.4171$. The standard error is $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/5594} \doteq 0.008372$, so the margin of error for 95% confidence is $1.96\,\text{SE}_{\hat{p}} \doteq 0.01641$, and the interval is 0.4007 to 0.4335. **(b)** The margin of error depends on the confidence level (95% in both cases) and on $\text{SE}_{\hat{p}}$. The sample size for male athletes was 1.6 times larger than the sample for female athletes, making the males' $\text{SE}_{\hat{p}}$ smaller, resulting in a smaller margin of error for men. (Another contributing factor to the smaller $\text{SE}_{\hat{p}}$ for males was that $\hat{p}(1-\hat{p})$ was slightly smaller for men: 0.2320 compared to 0.2431 for women.)

8.13. **(a)** $\text{SE}_{\hat{p}} = \sqrt{(0.87)(0.13)/430,000} \doteq 0.0005129$. For 99% confidence, the margin of error is $2.576\,\text{SE}_{\hat{p}} \doteq 0.001321$. **(b)** One source of error is indicated by the wide variation in response rates: We cannot assume that the statements of respondents represent the opinions of nonrespondents. The effect of the participation fee is harder to predict, but one possible impact is on the types of institutions that participate in the survey: Even though the fee is scaled for institution size, larger institutions can more easily absorb it. These other sources of error are much more significant than sampling error, which is the only error accounted for in the margin of error from part (a).

8.14. **(a)** The standard error is $\text{SE}_{\hat{p}} = \sqrt{(0.69)(0.31)/1048} \doteq 0.01429$, so the margin of error for 95% confidence is $1.96\,\text{SE}_{\hat{p}} \doteq 0.02800$ and the interval is 0.6620 to 0.7180. **(b)** To test H_0: $p = 0.79$ vs. H_a: $p < 0.79$, the standard error is $\sigma_{\hat{p}} = \sqrt{(0.79)(0.21)/1048} \doteq 0.01258$ and the test statistic is $z = \frac{0.69-0.79}{0.01258} \doteq -7.95$. This is very strong evidence against H_0 ($P < 0.00005$).

8.15. **(a)** The standard error is $\text{SE}_{\hat{p}} = \sqrt{(0.38)(0.62)/1048} \doteq 0.01499$, so the margin of error for 95% confidence is $1.96\,\text{SE}_{\hat{p}} \doteq 0.02939$ and the interval is 0.3506 to 0.4094. **(b)** Yes; some respondents might not admit to such behavior. The true frequency of such actions might be higher than this survey suggests.

8.16. **(a)** $\hat{p} = \frac{9054}{24{,}142} \doteq 0.3750$. **(b)** The standard error is $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/24{,}142} \doteq 0.003116$, so the margin of error for 95% confidence is $1.96\,\text{SE}_{\hat{p}} \doteq 0.00611$ and the interval is 0.3689 to 0.3811. **(c)** The nonresponse rate was $\frac{37{,}328 - 24{,}142}{37{,}328} \doteq 0.3532$. We have no way of knowing if cheating is more or less prevalent among nonrespondents; this weakens the conclusions we can draw from this survey.

8.17. **(a)** $\hat{p} = \frac{390}{1191} \doteq 0.3275$. The standard error is $\text{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/1191} \doteq 0.01360$, so the margin of error for 95% confidence is $1.96\,\text{SE}_{\hat{p}} \doteq 0.02665$ and the interval is 0.3008 to 0.3541. **(b)** Speakers and listeners probably perceive sermon length differently (just as, say, students and lecturers have different perceptions of the length of a class period).

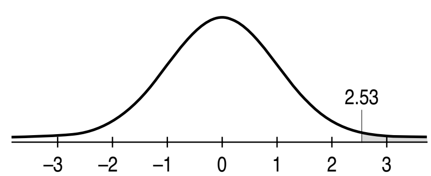
8.18. A 90% confidence interval would be narrower: The margin of error will be smaller (by a factor of 1.645/1.96) if we are willing to be less confident that we have included p. The large-sample 90% confidence interval is 0.3051 to 0.3498.

8.19. We estimate $\hat{p} = \frac{192}{1280} = 0.15$, $\text{SE}_{\hat{p}} \doteq 0.00998$, and the 95% confidence interval is 0.1304 to 0.1696.

8.20. A 99% confidence interval would be wider: We need a larger margin of error (by a factor of 2.576/1.96) in order to be more confident that we have included p. The large-sample 99% confidence interval is 0.1243 to 0.1757.

8.21. Recall the rule of thumb from Chapter 5: Use the Normal approximation if $np \geq 10$ and $n(1-p) \geq 10$. We use p_0 (the value specified in H_0) to make our decision.
(a) No: $np_0 = 6$. **(b)** Yes: $np_0 = 18$ and $n(1-p_0) = 12$. **(c)** Yes: $np_0 = n(1-p_0) = 50$. **(d)** No: $np_0 = 2$.

8.22. **(a)** Because we have defined p as the proportion who prefer fresh-brewed coffee, we should compute $\hat{p} = \frac{28}{40} = 0.7$. To test H_0: $p = 0.5$ vs. H_a: $p > 0.5$, the standard error is $\sigma_{\hat{p}} = \sqrt{(0.5)(0.5)/40} \doteq 0.07906$, and the test statistic is $z = \frac{0.7-0.5}{0.07906} \doteq 2.53$. The P-value is 0.0057. **(b)** Curve on the right. **(c)** The result is significant at the 5% level, so we reject H_0 and conclude that a majority of people prefer fresh-brewed coffee.



8.23. With $\hat{p} = 0.69$, $\text{SE}_{\hat{p}} \doteq 0.02830$ and the 95% confidence interval is 0.6345 to 0.7455. (If we assume that 184 of the 267 women had been on a diet, we could find the plus four interval: $\tilde{p} \doteq 0.6863$, $\text{SE}_{\tilde{p}} \doteq 0.02818$, and the interval is 0.6311 to 0.7416.)

8.24. With $\hat{p} = 0.583$, $\text{SE}_{\hat{p}} \doteq 0.03023$ and the 95% confidence interval is 0.5237 to 0.6423. (If we assume that 155 of the 266 high school students had been on a diet, we could find the plus four interval: $\tilde{p} \doteq 0.5815$, $\text{SE}_{\tilde{p}} \doteq 0.03002$, and the interval is 0.5226 to 0.6403.)

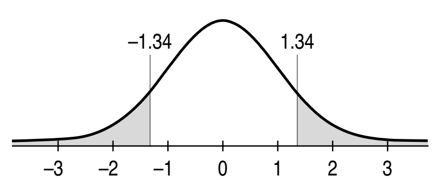
8.25. We estimate $\hat{p} = \frac{594}{2533} \doteq 0.2345$, $SE_{\hat{p}} \doteq 0.00842$, and the 95% confidence interval is 0.2180 to 0.2510. (The plus four method gives similar results: $\tilde{p} \doteq 0.2349$, $SE_{\tilde{p}} \doteq 0.00842$, and the 95% confidence interval is 0.2184 to 0.2514.)

8.26. **(a)** We estimate $\hat{p} = \frac{1434}{2533} \doteq 0.5661$, $SE_{\hat{p}} \doteq 0.00985$, and the 95% confidence interval is 0.5468 to 0.5854. (The plus four method gives similar results: $\tilde{p} \doteq 0.5660$, $SE_{\tilde{p}} \doteq 0.00984$, and the 95% confidence interval is 0.5467 to 0.5853.) **(b)** Pride or embarrassment might lead respondents to claim that their income was above $25,000 even if it were not. Consequently, it would not be surprising if the true proportion p were lower than the estimate $\hat{p}$. (There may also be some who would understate their income, out of humility or mistrust of the interviewer. While this would seem to have less of an impact, it makes it difficult to anticipate the overall effect of untruthful responses.) **(c)** Respondents would have little reason to lie about pet ownership; the few that might lie about it would have little impact on our conclusions. The number of untruthful responses about income is likely to be much larger and have a greater impact.

8.27. We estimate $\hat{p} = \frac{110}{125} = 0.88$, $SE_{\hat{p}} \doteq 0.02907$, and the 95% confidence interval is 0.8230 to 0.9370. (The plus four method gives similar results: $\tilde{p} \doteq 0.8682$, $SE_{\tilde{p}} \doteq 0.02978$, and the 95% confidence interval is 0.8098 to 0.9266.)

8.28. **(a)** $\hat{p} = \frac{542}{1711} \doteq 0.3168$; about 31.7% of bicyclists aged 15 or older killed between 1987 and 1991 had alcohol in their systems at the time of the accident. **(b)** $SE_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/1711} \doteq 0.01125$; the 99% confidence interval is $\hat{p} \pm 2.576\, SE_{\hat{p}} = 0.2878$ to 0.3457. (The plus four method gives similar results: $\tilde{p} \doteq 0.3172$, $SE_{\tilde{p}} \doteq 0.01124$, and the interval is 0.2883 to 0.3461.) **(c)** No: We do not know, for example, what percentage of cyclists who were *not* involved in fatal accidents had alcohol in their systems. **(d)** $\hat{p} = \frac{386}{1711} \doteq 0.2256$, $SE_{\hat{p}} \doteq 0.01010$, and the 99% confidence interval is 0.1996 to 0.2516. (The plus four method gives similar results: $\tilde{p} \doteq 0.2262$, $SE_{\tilde{p}} \doteq 0.01010$, and the interval is 0.2002 to 0.2523.)

8.29. **(a)** Testing H_0: $p = 0.5$ vs. H_a: $p \neq 0.5$, we have $\hat{p} = \frac{5067}{10000} = 0.5067$ and $\sigma_{\hat{p}} = \sqrt{(0.5)(0.5)/10000} = 0.005$, so $z = \frac{0.0067}{0.005} = 1.34$, for which $P = 0.1802$. This is not significant at $\alpha = 0.05$ (or even $\alpha = 0.10$). **(b)** $SE_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/10000} \doteq 0.005$, so the 95% confidence interval is $0.5067 \pm (1.96)(0.005)$, or 0.4969 to 0.5165.



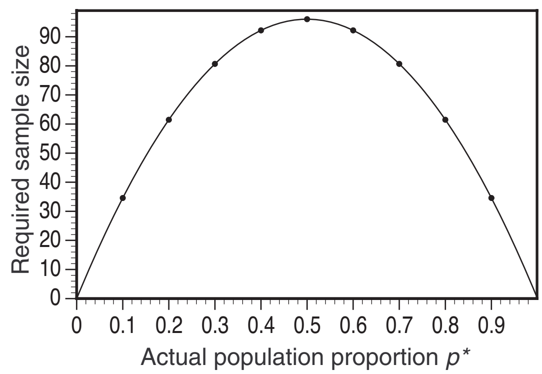
8.30. With no prior knowledge of the value of p (the proportion of "Yes" responses), take $p^* = 0.5$: $n = \left(\frac{1.96}{2(0.15)}\right)^2 \doteq 42.7$—use $n = 43$.

8.31. As a quick estimate, we can observe that to cut the margin of error in half, we must quadruple the sample size, from 43 to 172. Using the sample-size formula, we find $n = \left(\frac{1.96}{2(0.075)}\right)^2 \doteq 170.7$—use $n = 171$. (The difference in the two answers is due to rounding.)

8.32. Using $p^* = 0.25$ (based on previous surveys), we compute $n = \left(\frac{1.96}{0.1}\right)^2 (0.25)(0.75) \doteq 72.03$, so we need a sample of 73 students.

8.33. The required sample sizes are found by computing $\left(\frac{1.96}{0.1}\right)^2 p^*(1-p^*) = 384.16p^*(1-p^*)$: To be sure that we meet our target margin of error, we should take the largest sample indicated: $n = 97$ or larger.

p^*	n	Rounded up
0.1	34.57	35
0.2	61.47	62
0.3	80.67	81
0.4	92.20	93
0.5	96.04	97
0.6	92.20	93
0.7	80.67	81
0.8	61.47	62
0.9	34.57	35



8.34. $n = \left(\frac{1.96}{0.02}\right)^2 (0.15)(0.85) = 1224.51$—use $n = 1225$.

8.35. With $\hat{p}_w = \frac{45}{100} = 0.45$ and $\hat{p}_m = \frac{80}{140} \doteq 0.5714$, we estimate the difference to be $\hat{p}_m - \hat{p}_w \doteq 0.1214$. The standard error of the difference is $\text{SE}_D = \sqrt{\frac{\hat{p}_w(1-\hat{p}_w)}{100} + \frac{\hat{p}_m(1-\hat{p}_m)}{140}} \doteq 0.06499$, so the 95% confidence interval for $p_m - p_w$ is $0.1214 \pm (1.96)(0.06499) = -0.0060$ to 0.2488. (The plus four interval is -0.0069 to 0.2458.)

Note: *We followed the text's practice of subtracting the smaller proportion from the larger one, as described on the bottom of page 508.*

8.36. Let us call the proportions favoring Commercial B q_w and q_m. Our estimates of these proportions are the complements of those found in Exercise 8.35; for example, $\hat{q}_w = \frac{55}{100} = 0.55 = 1 - \hat{p}_w$. Consequently, the standard error of the difference $\hat{q}_w - \hat{q}_m$ is the same as that for $\hat{p}_m - \hat{p}_w$: $\text{SE}_D = \sqrt{\frac{\hat{q}_w(1-\hat{q}_w)}{100} + \frac{\hat{q}_m(1-\hat{q}_m)}{140}} \doteq 0.06499$. The margin of error is therefore also the same, and the 95% confidence interval for $q_w - q_m$ is $(\hat{q}_w - \hat{q}_m) \pm (1.96)(0.06499) = -0.0060$ to 0.2488.

Note: *As in the previous exercise, we followed the text's practice of subtracting the smaller proportion from the larger one.*

8.37. The pooled proportion $\hat{p} = \frac{45+80}{100+140} \doteq 0.5208$. For the test of H_0: $p_m = p_w$ vs. H_a: $p_m \neq p_w$, the appropriate standard error is $\text{SE}_{D_p} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{100} + \frac{1}{140}\right)} \doteq 0.06541$ and the test statistic is $z = (\hat{p}_m - \hat{p}_w)/\text{SE}_{D_p} \doteq 1.86$, for which the two-sided P-value is 0.0629. This is not quite enough evidence to reject H_0 at the 5% level.

8.38. Because the sample proportions would tend to support the alternative hypothesis ($p_m > p_w$), the P-value is half as large ($P = 0.0314$), which would be enough to reject H_0 at the 5% level.

8.39. Recall the text's guidelines: Large-sample intervals can be used when the number of successes and the number of failures in both samples are all at least 10. **(a)** Yes: The smallest count is 10. **(b)** No: Three of the four counts are only 5. **(c)** No: One count is only 8. **(d)** Yes: The smallest count is 12. **(e)** No: One count is only 5.

Note: *The plus four interval is recommended when both sample sizes are at least 5, so it could be used in all cases.*

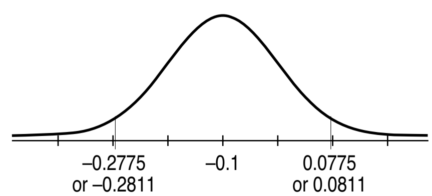
8.40. The guidelines are summarized in the previous solution. **(a)** No: One count is 8. **(b)** No: One count is 8. **(c)** Yes: All counts are at least 15. **(d)** Yes: All counts are at least 15. **(e)** No: One count is 6.

Note: *The plus four interval could be used in all of these cases.*

8.41. Recall that in Exercise 8.4, we suspected cell phone ownership had increased, so we use a one-sided alternative: H_0: $p_{2003} = p_{2004}$ vs. H_a: $p_{2003} < p_{2004}$. With $\hat{p}_{2003} = 0.83$ and $\hat{p}_{2004} = 0.89$, the pooled proportion is $\hat{p} = 0.86$ (the average of the two sample proportions, since the sample sizes are equal). The standard error for this test is $\mathrm{SE}_{D_p} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{1200} + \frac{1}{1200}\right)} \doteq 0.1417$ and the test statistic is $z = (\hat{p}_{2004} - \hat{p}_{2003})/\mathrm{SE}_{D_p} \doteq 4.24$, for which $P < 0.0001$. This is strong evidence that cell phone ownership has increased.

To construct a 95% confidence interval for $p_{2004} - p_{2003}$ (the increase in cell phone ownership), the appropriate standard error is $\mathrm{SE}_D = \sqrt{\frac{\hat{p}_{2003}(1-\hat{p}_{2003})}{1200} + \frac{\hat{p}_{2004}(1-\hat{p}_{2004})}{1200}} \doteq 0.01411$ and the interval is $0.06 \pm (1.96)(0.01411) = 0.0323$ to 0.0877. (The plus four interval is 0.0322 to 0.0876.)

8.42. **(a)** The mean is $\mu_D = p_1 - p_2 = 0.3 - 0.4 = -0.1$, and the standard deviation is $\sigma_D = \sqrt{\frac{p_1(1-p_1)}{50} + \frac{p_2(1-p_2)}{60}} \doteq 0.09055$. **(b)** Normal curve on the right. **(c)** d should be either $1.96\sigma_D \doteq 0.1775$ or $2\sigma_D \doteq 0.1811$, so the points marked on the curve should be either -0.2775 and 0.0775 or -0.2811 and 0.0811.



Note: *Because this problem told us which population was "first" and which was "second," we did not follow the suggestion in the text to arrange them so that the population 1 had the larger proportion. Where necessary, we have done so in the other exercises.*

8.43. We have $\hat{p}_m = \frac{3547}{5594} \doteq 0.6341$, $\hat{p}_f = \frac{1447}{3469} \doteq 0.4171$, and pooled proportion $\hat{p} = \frac{3547+1447}{5594+3469} \doteq 0.5510$. For the test of H_0: $p_m = p_f$ vs. H_a: $p_m \neq p_f$, the appropriate standard error is $\mathrm{SE}_{D_p} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{5594} + \frac{1}{3469}\right)} \doteq 0.01075$ and the test statistic is $z = (\hat{p}_m - \hat{p}_f)/\mathrm{SE}_{D_p} \doteq 20.18$—overwhelming evidence that the two proportions are different. The standard error for a confidence interval is

$SE_D = \sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2} \doteq 0.01056$, and the 95% confidence interval is 0.1962 to 0.2377. (The plus four interval is nearly identical: 0.1962 to 0.2376.) This interval and the significance test are only designed to deal with random sampling error; other sources of error such as nonresponse could throw all conclusions into doubt.

8.44. Pet owners had the lower proportion of women, so we call them "population 2": $\hat{p}_2 = \frac{285}{595} \doteq 0.4790$. For non-pet owners, $\hat{p}_1 = \frac{1024}{1939} \doteq 0.5281$. $SE_D \doteq 0.02341$, so the 95% confidence interval is 0.0032 to 0.0950. (The plus four interval is 0.0032 to 0.0948.)

8.45. Population 1 is non-pet owners: $\hat{p}_1 = 0.577$ and $\hat{p}_2 = 0.533$, so $SE_D \doteq 0.02333$ and the 95% confidence interval is -0.0017 to 0.0897. (If we assume that 1119 non-pet owners and 317 pet owners were married, then the plus four interval is -0.0013 to 0.0900.) The proportion of non-pet owners who are married is 0% to 9% higher than that proportion for pet owners.

8.46. To test H_0: $p_{1993} = p_{1999}$ vs. H_a: $p_{1993} \neq p_{1999}$, we have $\hat{p}_{1993} \doteq 0.1983$ and $\hat{p}_{1999} \doteq 0.2272$, pooled proportion $\hat{p} \doteq 0.2122$, and $SE_{D_p} \doteq 0.00482$. The test statistic $z \doteq 6.01$ gives strong evidence ($P < 0.0001$) that the proportion of frequent binge drinkers was higher in 1999.

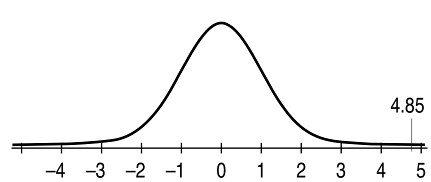
To construct a 95% confidence interval for the proportion $p_{1999} - p_{1993}$, we find $SE_D \doteq 0.00483$; the interval is 0.0195 to 0.0384.

The two surveys give strong evidence that the 1999 proportion is higher, but the increase is fairly modest: only 2 to 3.8 percentage points. (The tiny P-value arises because we have large sample sizes.)

8.47. Let us call the proportions of nonfrequent binge q_{1993} and q_{1999}. Our estimates of these proportions are the complements of those found in Exercise 8.46; for example, $\hat{q}_{1993} = \frac{12{,}022}{14{,}995} \doteq 0.8017 = 1 - \hat{p}_{1993}$. Likewise, the pooled proportion $\hat{q} \doteq 0.7878$ is the complement of $\hat{p}$ from that exercise. Consequently, both standard errors (for the test and the interval) are unchanged: $SE_{D_p} \doteq 0.00482$ and $SE_D \doteq 0.00483$. The test statistic ($z \doteq 6.01$), P-value ($P < 0.0001$), and interval (0.0195 to 0.0384) are therefore the same as before.

Note: *See also the solutions to Exercises 8.35 and 8.36 for a similar comparison.*

8.48. **(a)** $\hat{p}_1 = \frac{35}{165} \doteq 0.2121$ and $\hat{p}_2 = \frac{17}{283} \doteq 0.0601$ (arranged so that population 1 has the larger proportion). **(b)** $\hat{p}_1 - \hat{p}_2 \doteq 0.1521$ and the standard error (for constructing a confidence interval) is $SE_D \doteq 0.03482$. **(c)** The hypotheses are H_0: $p_1 = p_2$ vs. H_a: $p_1 > p_2$.



The alternative reflects the reasonable expectation that reducing pollution might decrease wheezing. **(d)** The pooled estimate of the proportion is $\hat{p} = \frac{17+35}{283+165} \doteq 0.1161$ and $SE_{D_p} \doteq 0.03137$, so $z = (\hat{p}_1 - \hat{p}_2)/SE_{D_p} \doteq 4.85$. The P-value is very small ($P < 0.0001$). **(e)** The 95% confidence interval, using the standard error from part (b), has margin of error $1.96\,SE_D \doteq 0.06824$: 0.0838 to 0.2203. (The plus four interval is 0.0839 to 0.2209.) The percentage reporting improvement was between 8% and 22% higher for bypass residents. **(f)** There may be geographic factors (e.g., weather) or cultural factors (e.g., diet) that limit how much we can generalize the conclusions.

8.49. With equal sample sizes, the pooled estimate of the proportion is $\hat{p} = 0.255$, the average of $\hat{p}_1 = 0.29$ and $\hat{p}_2 = 0.22$. This can also be computed by taking $X_1 = (0.29)(1421) = 412.09$ and $X_2 = (0.22)(1421) = 312.62$, so $\hat{p} = (X_1 + X_2)/(1421 + 1421)$. The standard error for a significance test is $\text{SE}_{D_p} \doteq 0.01635$, and the test statistic is $z \doteq 4.28$ ($P < 0.0001$); we conclude that the proportions are different. The standard error for a confidence interval is $\text{SE}_D \doteq 0.01630$, and the 95% confidence interval is 0.0381 to 0.1019. The interval gives us an idea of how large the difference is: Music downloads dropped 4% to 10%.

8.50. The table below shows the results from the previous exercise, and those with different sample sizes. For part (iii), two answers are given, corresponding to the two ways one could interpret which is the "first sample size."

	n_1	n_2	$\hat{p}$	SE_{D_p}	z	SE_D	Confidence interval
8.50	1421	1421	0.255	0.01635	4.28	0.01630	0.0381 to 0.1019
(i)	1000	1000	0.255	0.01949	3.59	0.01943	0.0319 to 0.1081
(ii)	1600	1600	0.255	0.01541	4.54	0.01536	0.0399 to 0.1001
(iii)	1000	1600	0.2469	0.01738	4.03	0.01770	0.0353 to 0.1047
	1600	1000	0.2631	0.01775	3.94	0.01733	0.0360 to 0.1040

As one would expect, we see in (i) and (ii) that smaller samples result in smaller z (weaker evidence) and wider intervals, while larger samples have the reverse effect. The results of (iii) show that the effect of varying unequal sample sizes is more complicated.

8.51. **(a)** We find $\hat{p}_1 = \frac{73}{91} \doteq 0.8022$ and $\hat{p}_2 = \frac{75}{109} \doteq 0.6881$. For a confidence interval, $\text{SE}_D \doteq 0.06093$, so the 95% confidence interval for $p_1 - p_2$ is $(0.8022 - 0.6881) \pm (1.96)(0.06093) = -0.0053$ to 0.2335. (The plus four interval is -0.0081 to 0.2301.) **(b)** The question posed was, "Do high-tech companies tend to offer stock options more often than other companies?" Therefore, we test H_0: $p_1 = p_2$ vs. H_a: $p_1 > p_2$. With $\hat{p}_1 \doteq 0.8022$, $\hat{p}_2 \doteq 0.6881$, and $\hat{p} = \frac{73+75}{91+109} = 0.74$, we find $\text{SE}_{D_p} \doteq 0.06229$, so $z = (\hat{p}_1 - \hat{p}_2)/\text{SE}_{D_p} \doteq 1.83$. This gives $P = 0.0336$. **(c)** We have fairly strong evidence that high-tech companies are more likely to offer stock options. However, the confidence interval tells us that the difference in proportions could be very small, or as large as 23%.

8.52. With $\hat{p}_{2002} \doteq 0.4780$ and $\hat{p}_{2004} \doteq 0.3750$, the standard error for a confidence interval is $\text{SE}_D \doteq 0.00550$. The 90% confidence interval for the difference $p_{2002} - p_{2004}$ is $(0.4780 - 0.3750) \pm 1.645\text{SE}_D = 0.0939$ to 0.1120.

8.53. **(a)** $\hat{p}_\text{f} = \frac{48}{60} = 0.8$, so $\text{SE}_{\hat{p}} \doteq 0.05164$ for females. $\hat{p}_\text{m} = \frac{52}{132} = 0.\overline{39}$, so $\text{SE}_{\hat{p}} \doteq 0.04253$ for males. **(b)** $\text{SE}_D = \sqrt{0.05164^2 + 0.04253^2} \doteq 0.06690$, so the interval is $(\hat{p}_\text{f} - \hat{p}_\text{m}) \pm 1.645\,\text{SE}_D$, or 0.2960 to 0.5161. There is (with high confidence) a considerably higher percentage of juvenile references to females than to males.

8.54. **(a)** $\hat{p}_1 = \frac{515}{1520} \doteq 0.3388$ for men, and $\hat{p}_2 = \frac{27}{191} \doteq 0.1414$ for women. $\text{SE}_D \doteq 0.02798$, so the 95% confidence interval for $p_1 - p_2$ is 0.1426 to 0.2523. (The plus four interval is 0.1389 to 0.2490.) **(b)** The female contribution is larger because the sample size for women is much smaller. (Specifically, $\hat{p}_1(1-\hat{p}_1)/n_1 \doteq 0.0001474$, while $\hat{p}_2(1-\hat{p}_2)/n_2 \doteq 0.0006355$.) Note that if the sample sizes had been similar, the male contribution would have been larger (assuming the proportions remained the same) because the numerator term $p_i(1-p_i)$ is greater for men than women.

8.55. We test H_0: $p_1 = p_2$ vs. H_a: $p_1 \neq p_2$. With $\hat{p}_1 \doteq 0.5281$, $\hat{p}_2 \doteq 0.4790$, and $\hat{p} = \frac{1024+285}{1939+595} \doteq 0.5166$, we find $\text{SE}_{D_p} \doteq 0.02342$, so $z = (\hat{p}_1 - \hat{p}_2)/\text{SE}_{D_p} \doteq 2.10$. This gives $P = 0.0360$—significant evidence (at the 5% level) that a higher proportion of non-pet owners are women.

8.56. We test H_0: $p_1 = p_2$ vs. H_a: $p_1 \neq p_2$. We were given $\hat{p}_1 = 0.577$ and $\hat{p}_2 = 0.533$, which correspond to $X_1 \doteq 1119^*$ and $X_2 = 317$. Then, $\hat{p} = \frac{1119+317}{1939+595} \doteq 0.5667$, $\text{SE}_{D_p} \doteq 0.02322$, $z = (\hat{p}_1 - \hat{p}_2)/\text{SE}_{D_p} \doteq 1.91$, and $P = 0.0563$. Although the proportions suggest that pet owners are more likely to be married, the difference is not quite significant.

*In fact, either $X_1 = 1118$ or 1119 would result in $\hat{p}_1 = 0.577$ when rounded to three decimal places. The impact of choosing $X_1 = 1118$, or of using $X_1 = (1939)(0.577) = 1118.803$ and $X_2 = (595)(0.533) = 317.135$, is minimal; either approach results in $z \doteq 1.89$ instead of 1.91, and a very similar P-value.

8.57. **(a)** Confidence intervals only account for random sampling error. **(b)** H_0 should refer to p_1 and p_2 (population proportions) rather than $\hat{p}_1$ and $\hat{p}_2$ (sample proportions). **(c)** Knowing $\hat{p}_1 = \hat{p}_2$ does not tell us that the success counts are equal ($X_1 = X_2$) *unless* the sample sizes are equal ($n_1 = n_2$).

8.58. With $\hat{p} = 0.58$, the standard error is $\text{SE}_{\hat{p}} = \sqrt{(0.58)(0.42)/1048} \doteq 0.01525$, so the margin of error for 90% confidence is $1.645\,\text{SE}_{\hat{p}} \doteq 0.02508$, and the interval is 0.5549 to 0.6051.

8.59. With $\hat{p}_m = 0.59$ and $\hat{p}_w = 0.56$, the standard error is $\text{SE}_D \doteq 0.03053$, the margin of error for 95% confidence is $1.96\text{SE}_D \doteq 0.05983$, and the confidence interval for $p_m - p_w$ is -0.0298 to 0.0898.

8.60. **(a)** The table below summarizes the margins of error m.e.$= 1.96\sqrt{\hat{p}(1-\hat{p})/n}$:

		$\hat{p}$	m.e.	95% confidence interval
Current	Downloading less	38%	6.05%	31.95% to 44.05%
downloaders	Use P2P networks	33.33%	5.88%	27.45% to 39.21%
($n = 247$)	Use e-mail/IM	24%	5.33%	18.67% to 29.33%
	Use music-related sites	20%	4.99%	15.01% to 24.99%
	Use paid services	17%	4.68%	12.32% to 21.68%
All users	Have used paid services	7%	1.35%	5.65% to 8.35%
($n = 1371$)	Currently use paid services	3%	0.90%	2.10% to 3.90%

(b) Obviously, students' renditions of the above information in a paragraph will vary.
(c) Student opinions may vary on this. Personally, I lean toward B, although I would be inclined to report two margins of error: "no more than 6%" for the current downloaders and "no more than 1.4%" for all users.

8.61. We compute $\hat{p} = \frac{152}{248} \doteq 0.6129$. To test H_0: $p = 0.485$ vs. H_a: $p \neq 0.485$, the standard error is $\sigma_{\hat{p}} = \sqrt{(0.485)(0.515)/248} \doteq 0.03174$, the test statistic is $z = (\hat{p} - 0.485)/\sigma_{\hat{p}} \doteq 4.03$, and $P < 0.0001$. For constructing a confidence interval, the appropriate standard error is $\mathrm{SE}_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/248} \doteq 0.03093$, so the 95% confidence interval is 0.5523 to 0.6735. (The plus four method gives similar results: $\tilde{p} \doteq 0.6111$, $\mathrm{SE}_{\tilde{p}} \doteq 0.03071$, and the interval is 0.5509 to 0.6713.) The significance test revealed strong evidence that heavy lottery players are more likely to be men; the confidence interval further tells us that between 55% and 67% of heavy lottery players are men.

8.62. (a) With $\hat{p}_1 = 0.89$ and $\hat{p}_2 = 0.43$, $\hat{p}_1 - 2\hat{p}_2 = 0.03$. **(b)** Assuming that both sample sizes were 1200, the standard error of $\hat{p}_i$ is $\sqrt{\hat{p}_i(1-\hat{p}_i)/1200}$, so $\mathrm{SE}_{\hat{p}_1} \doteq 0.009032$ and $\mathrm{SE}_{\hat{p}_2} = 0.014292$. Note that $\hat{p}_1$ and $\hat{p}_2$ are independent, so by rules 1 and 2 from page 282, we see that the estimated variance for $\hat{p}_1 - 2\hat{p}_2$ is $\mathrm{SE}^2_{\hat{p}_1} + 4\mathrm{SE}^2_{\hat{p}_2} \doteq 0.00089858$. The standard error is the square root of this value: $\mathrm{SE}_{\hat{p}_1 - 2\hat{p}_2} \doteq 0.02998$. **(c)** To test H_0: $p_1 = 2p_2$ vs. H_a: $p_1 > 2p_2$, the z statistic is essentially (see the note that follows) the estimate from (a) divided by the standard error from (b): $z \doteq 1.00$, for which $P \doteq 0.1587$. This is not enough evidence to reject H_0.

Note: *The standard error computed in (b) is appropriate for a confidence interval, but not for a hypothesis test. For testing H_0: $p_1 = p_2$, we use a pooled estimate of the common proportion. In this case, the corresponding idea would be to compute pooled estimates of p_1 and p_2 working with the assumption that $p_1 = 2p_2$. For example, as pooled estimates for p_1 and p_2, we could use $\widehat{p_1} = \frac{X_1 + 2X_2}{n_1 + n_2} = \frac{1}{2}(\hat{p}_1 + 2\hat{p}_2) = 0.875$ and $\widehat{p_2} = \frac{1}{2}\widehat{p_1} = 0.4375$. With these values, the standard errors would be $\mathrm{SE}_{\hat{p}_1} \doteq 0.009547$, $\mathrm{SE}_{\hat{p}_2} \doteq 0.014321$, and $\mathrm{SE}_{\hat{p}_1 - 2\hat{p}_2} \doteq 0.03019$. We now see that this more complicated method leads to the same conclusion: $z \doteq 0.99$ and $P = 0.1611$.*

Noting that the two populations (2000 undergraduate students and 2004 undergraduate students) might have some overlap, one might question whether the two samples are necessarily independent. Given the size of the two populations, and the relatively small size of that overlap, the two sample proportions will be close enough to independence that the methods of this chapter should be safe.

8.63. (a) People have different symptoms; for example, not all who wheeze consult a doctor. **(b)** In the table (below), we find for "sleep" that $\hat{p}_1 = \frac{45}{282} \doteq 0.1596$ and $\hat{p}_2 = \frac{12}{164} \doteq 0.0732$, so the difference is $\hat{p}_1 - \hat{p}_2 \doteq 0.0864$. Therefore, $\mathrm{SE}_D \doteq 0.02982$ and the margin of error for 95% confidence is 0.05844. Other computations are performed in like manner. (Plus four intervals are no shown, but are similar.) **(c)** It is reasonable to expect that the bypass proportions would be higher—that is, we expect more improvement where the pollution decreased—so we could use the alternative $p_1 > p_2$. **(d)** For "sleep," we find $\hat{p} = \frac{45+12}{282+164} \doteq 0.1278$ and $\mathrm{SE}_{D_p} \doteq 0.03279$. Therefore, $z \doteq (0.1596 - 0.0732)/\mathrm{SE}_{D_p} \doteq 2.64$. Other computations are similar. Only the "sleep" difference is significant. **(e)** 95% confidence intervals are shown below. Part (b) showed improvement relative to control group, which is a better measure of the effect of the bypass, because it allows us to account for the improvement reported over time even when no change was made.

	Bypass minus congested				Bypass	
Complaint	$\hat{p}_1 - \hat{p}_2$	95% CI	z	P	$\hat{p}$	95% CI
Sleep	0.0864	0.0280 to 0.1448	2.64	0.0042	0.1596	0.1168 to 0.2023
Number	0.0307	−0.0361 to 0.0976	0.88	0.1897	0.1596	0.1168 to 0.2023
Speech	0.0182	−0.0152 to 0.0515	0.99	0.1600	0.0426	0.0190 to 0.0661
Activities	0.0137	−0.0395 to 0.0670	0.50	0.3100	0.0925	0.0586 to 0.1264
Doctor	−0.0112	−0.0796 to 0.0573	−0.32	0.6267	0.1174	0.0773 to 0.1576
Phlegm	−0.0220	−0.0711 to 0.0271	−0.92	0.8217	0.0474	0.0212 to 0.0736
Cough	−0.0323	−0.0853 to 0.0207	−1.25	0.8950	0.0575	0.0292 to 0.0857

8.64. **(a)** $\hat{p}_\text{f} = \frac{63}{296} \doteq 0.2128$ and $\hat{p}_\text{m} = \frac{27}{251} \doteq 0.1076$, so $\text{SE}_D \doteq 0.03080$ and the interval is 0.0449 to 0.1656. (The plus four interval is 0.0435 to 0.1647.) **(b)** For testing H_0: $p_\text{f} = p_\text{m}$ vs. H_a: $p_\text{f} \neq p_\text{m}$, we find $\hat{p} = \frac{63+27}{296+251} \doteq 0.1645$, we find $\text{SE}_{D_p} \doteq 0.03181$, so $z = (\hat{p}_\text{f} - \hat{p}_\text{m})/\text{SE}_{D_p} \doteq 3.31$. This gives $P = 0.0009$—significant evidence that women are more likely than men to be label users.

8.65. **(a)** For testing H_0: $p_1 = p_2$ vs. H_a: $p_1 \neq p_2$, we find $\hat{p}_1 = \frac{643}{1132} \doteq 0.5680$, $\hat{p}_2 = \frac{349}{852} \doteq 0.4096$, and $\hat{p} = \frac{643+349}{1132+852} = 0.5$. Then $\text{SE}_{D_p} \doteq 0.02268$, so $z = (\hat{p}_1 - \hat{p}_2)/\text{SE}_{D_p} \doteq 6.98$. This gives a tiny P-value and very strong evidence that those who arrange travel on the Internet are more likely to have completed college. **(b)** $\text{SE}_D \doteq 0.02237$ and the 95% confidence interval is 0.1145 to 0.2022. (The plus four interval is 0.1143 to 0.2019.)

8.66. We choose to look at the proportions with income over \$50,000; the results are essentially the same if we work with the complementary proportions. The sample proportions are $\hat{p}_1 = \frac{378}{871} \doteq 0.4340$ and $\hat{p}_2 = \frac{200}{677} \doteq 0.2954$; the pooled proportion for testing H_0: $p_1 = p_2$ vs. H_a: $p_1 \neq p_2$ is $\hat{p} = \frac{378+200}{871+677} \doteq 0.3734$. This leads to $\text{SE}_{D_p} \doteq 0.02478$, $z \doteq 5.59$, and $P < 0.0001$. For the 95% confidence interval for $p_1 - p_2$, $\text{SE}_D \doteq 0.02428$, so the interval is 0.0910 to 0.1861 (the plus four interval is 0.0906 to 0.1857).

8.67. For the education question, there were 1132 users and 852 nonusers. Only 871 users and 677 nonusers responded to the income question, so the proportions not responding to the income question were (for users) $\hat{p}_1 = \frac{1132-871}{1132} \doteq 0.2306$ and (for nonusers) $\hat{p}_2 = \frac{852-677}{852} \doteq 0.2054$. Therefore, $\hat{p} \doteq 0.2198$, $\text{SE}_{D_p} \doteq 0.01878$, and $z \doteq 1.34$. For a two-sided alternative, $P = 0.1802$, so we have little reason to suspect a difference in nonresponse rates between users and nonusers. For a 95% confidence interval, $\text{SE}_D \doteq 0.01866$ and the interval is −0.0114 to 0.0617 (the plus four interval is −0.0116 to 0.0615).

More than 20% did not respond to the income question; this lack of response makes the conclusions for Exercise 8.66 suspect.

8.68. (a) The number of orders completed in five days or less before the changes was $X_1 = (0.16)(200) = 32$. With $\hat{p}_1 = 0.16$, $\text{SE}_{\hat{p}} \doteq 0.02592$, and the 95% confidence interval for p_1 is 0.1092 to 0.2108. (The plus four interval is 0.1155 to 0.2178.) **(b)** After the changes, $X_2 = (0.9)(200) = 180$. With $\hat{p}_2 = 0.9$, $\text{SE}_{\hat{p}} \doteq 0.02121$, and the 95% confidence interval for p_2 is 0.8584 to 0.9416. (The plus four interval is 0.8496 to 0.9347.) **(c)** $\text{SE}_D \doteq 0.03350$ and the 95% confidence interval for $p_2 - p_1$ is 0.6743 to 0.8057, or about 67.4% to 80.6%. (The plus four interval is 0.6666 to 0.7988.)

8.69. With $\hat{p} = 0.56$, $\text{SE}_{\hat{p}} \doteq 0.01433$, so the margin of error for 95% confidence is $1.96\text{SE}_{\hat{p}} \doteq 0.02809$.

8.70. (a) $X_1 = 121 \doteq (0.903)(134)$ die-hard fans and $X_2 = 161 \doteq (0.679)(237)$ less loyal fans watched or listened as children. **(b)** $\hat{p} = \frac{121+161}{134+237} \doteq 0.7601$ and $\text{SE}_{D_p} \doteq 0.04615$, so we find $z \doteq 4.85$ ($P < 0.0001$)—strong evidence of a difference in childhood experience. **(c)** For a 95% confidence interval, $\text{SE}_D \doteq 0.03966$ and the interval is 0.1459 to 0.3014. (The plus four interval is 0.1410 to 0.2975.) If students work with the rounded proportions (0.903 and 0.679), the 95% confidence interval is 0.1463 to 0.3017.

8.71. With $\hat{p}_1 = \frac{2}{3}$ and $\hat{p}_2 = 0.2$, we have $\hat{p} = \frac{134\hat{p}_1+237\hat{p}_2}{134+237} \doteq 0.3686$, $\text{SE}_{D_p} \doteq 0.05214$, and $z = 8.95$—very strong evidence of a difference. (If we assume that "two-thirds of the die-hard fans" and "20% of the less loyal fans" mean 89 and 47 fans respectively, then $\hat{p} \doteq 0.3666$ and $z \doteq 8.94$; the conclusion is the same.) For a 95% confidence interval, $\text{SE}_D \doteq 0.04831$ and the interval is 0.3720 to 0.5613. (With $X_1 = 89$ and $X_2 = 47$, the interval is 0.3712 to 0.5606; the plus four interval is 0.3666 to 0.5553.)

8.72. We test H_0: $p_f = p_m$ vs. H_a: $p_f \neq p_m$ for each text, where, for example, p_f is the proportion of juvenile female references. We can reject H_0 for texts 2, 3, 6, and 10. The last three texts do not stand out as different from the first seven. Texts 7 and 9 are notable as the only two with a majority of juvenile male references, while six of the ten texts had juvenile female references a majority of the time.

Text	$\hat{p}_f$	$\hat{p}_m$	$\hat{p}$	z	P
1	.4000	.2059	.2308	0.96	.3361
2	.7143	.2857	.3286	2.29	.0220
3	.4464	.2154	.3223	2.71	.0067
4	.1447	.1210	.1288	0.51	.6123
5	.6667	.2791	.3043	1.41	.1584
6	.8000	.3939	.5208	5.22	.0000
7	.9500	.9722	.9643	−0.61	.5437
8	.2778	.1818	.2157	0.80	.4259
9	.6667	.7273	.7097	−0.95	.3399
10	.7222	.2520	.3103	4.04	.0001

8.73. The proportions, z-values, and P-values are:

Text	1	2	3	4	5	6	7	8	9	10
$\hat{p}$	.8718	.9000	.5372	.6738	.9348	.6875	.6429	.6471	.7097	.8759
z	4.64	6.69	0.82	5.31	5.90	5.20	3.02	2.10	6.60	9.05
P	≈ 0	≈ 0	.4133	≈ 0	≈ 0	≈ 0	.0025	.0357	≈ 0	≈ 0

We reject H_0: $p = 0.5$ for all texts except Text 3 and (perhaps) Text 8. If we are using a "multiple comparisons" procedure such as Bonferroni (see Chapter 6), we also might fail to reject H_0 for Text 7.

The last three texts do not seem to be any different from the first seven; the gender of the author does not seem to affect the proportion.

8.74. **(a)** $\hat{p} = \frac{463}{975} \doteq 0.4749$, $\text{SE}_D \doteq 0.01599$, and the 95% confidence interval is 0.4435 to 0.5062. (The plus four interval is 0.4437 to 0.5063.) **(b)** Expressed as percents, the confidence interval is 44.35% to 50.62% (plus four: 44.37% to 50.63%). **(c)** Multiply the upper and lower limits of the confidence interval by 37,500: about 16,632 to 18,983 students (plus four: 16,638 to 18,985 students).

8.76. With sample sizes of $n_w = (0.52)(1200) = 624$ women and $n_m = 576$ men, we test H_0: $p_m = p_w$ vs. H_a: $p_m \neq p_w$. Assuming there were $X_m = 0.62n_m = 357.12$ men and $X_w = 0.51n_w = 318.24$ women who thought that parents put too little pressure on students, the pooled estimate is $\hat{p} \doteq 0.5628$, $\text{SE}_{D_p} \doteq 0.02866$, and the test statistic is $z \doteq 3.84$. This is strong evidence ($P < 0.0001$) that a higher proportion of men have this opinion.

To construct a 95% confidence interval for $p_m - p_w$, we have $\text{SE}_D \doteq 0.02845$, yielding the interval 0.0542 to 0.1658.

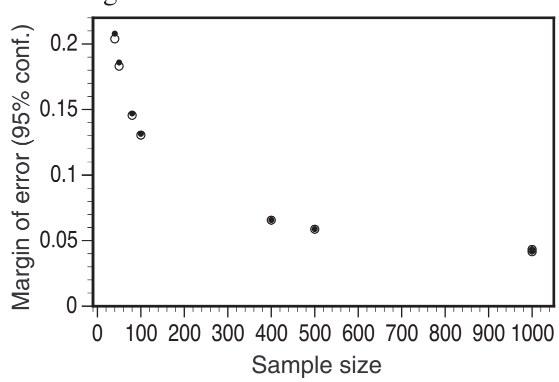
8.77. The difference becomes more significant (i.e., the P-value decreases) as the sample size increases. For small sample sizes, the difference between $\hat{p}_1 = 0.5$ and $\hat{p}_2 = 0.4$ is not significant, but with larger sample sizes, we expect that the sample proportions should be better estimates of their respective population proportions, so $\hat{p}_1 - \hat{p}_2 = 0.1$ suggests that $p_1 \neq p_2$.

n	z	P
40	0.90	0.3681
50	1.01	0.3125
80	1.27	0.2041
100	1.42	0.1556
400	2.84	0.0045
500	3.18	0.0015
1000	4.49	0.0000

8.78. Shown are both the large sample (m.e.$_{\text{large}}$, "•" in the graph) and plus four (m.e.$_{+4}$, "○" in the graph) margins of error. The graph illustrates how the two methods give very similar answers for $n \geq 50$. (Unless otherwise instructed, students will presumably only give the large-sample margins of error.)

As we observed with a single proportion, the margin of error decreases as sample size increases, but the rate of decrease is notably less for large n.

n	m.e.$_{\text{large}}$	m.e.$_{+4}$
40	0.2079	0.2039
50	0.1859	0.1831
80	0.1470	0.1456
100	0.1315	0.1305
400	0.0657	0.0656
500	0.0588	0.0587
1000	0.0416	0.0415



8.79. **(a)** Using either trial and error, or the formula derived in part (b), we find that at least $n = 342$ is needed. **(b)** Generally, the margin of error is $m = z^*\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n} + \frac{\hat{p}_2(1-\hat{p}_2)}{n}}$; with $\hat{p}_1 = \hat{p}_2 = 0.5$, this is $m = z^*\sqrt{0.5/n}$. Solving for n, we find $n = (z^*/m)^2/2$.

8.80. We must assume that we can treat the births recorded during these two times as independent SRSs. Note that the rules of thumb for the Normal approximation are not satisfied here; specifically, three birth defects are less than ten. Additionally, one might call into question the assumption of independence, because there may have been multiple births to the same set of parents included in these counts (either twins/triplets/etc., or "ordinary" siblings).

If we carry out the analysis in spite of these issues, we find $\hat{p}_1 = \frac{16}{414} \doteq 0.03865$ and $\hat{p}_2 = \frac{3}{228} \doteq 0.01316$. We might then find a 95% confidence interval: $\text{SE}_D \doteq 0.01211$, so the interval is $\hat{p}_1 - \hat{p}_2 \pm (1.96)(0.01211) = 0.00175$ to 0.04923. (Note that this does not take into account the presumed direction of the difference.)

This setting does meet our requirements for the plus four method, for which $\tilde{p}_1 = 0.04086$ and $\tilde{p}_2 = 0.01739$, $\text{SE}_D = 0.01298$, and the 95% confidence interval is -0.0020 to 0.0489.

We could also perform a significance test of H_0: $p_1 = p_2$ vs. H_a: $p_1 > p_2$: $\hat{p} = \frac{19}{642} \doteq 0.02960$, $\text{SE}_{D_p} \doteq 0.01398$, $z \doteq 1.82$, $P = 0.0344$.

Both the large-sample interval and the significance test suggest that the two proportions are different (but not much); the plus four interval does not establish that $p_1 \neq p_2$. Also, we must recognize that the issues noted above make this conclusion questionable.

8.81. **(a)** $p_0 = \frac{143{,}611}{181{,}535} \doteq 0.7911$. **(b)** $\hat{p} = \frac{339}{870} \doteq 0.3897$, $\sigma_{\hat{p}} \doteq 0.0138$, and $z = (\hat{p} - p_0)/\sigma_{\hat{p}} \doteq -29.1$, so $P \doteq 0$ (regardless of whether H_a is $p > p_0$ or $p \neq p_0$). This is very strong evidence against H_0; we conclude that Mexican Americans are underrepresented on juries. **(c)** $\hat{p}_1 = \frac{339}{870} \doteq 0.3897$, while $\hat{p}_2 = \frac{143{,}611 - 339}{181{,}535 - 870} \doteq 0.7930$. Then $\hat{p} \doteq 0.7911$ (the value of p_0 from part (a)), $\text{SE}_{D_p} \doteq 0.01382$, and $z \doteq -29.2$—and again, we have a tiny P-value and reject H_0.

8.83. In each case, the standard error is $\sqrt{\hat{p}(1-\hat{p})/1280}$. One observation is that, while many feel that loans are a burden and wish they had borrowed less, a majority are satisfied with the benefits they receive from their education.

	$\hat{p}$	$\text{SE}_{\hat{p}}$	95% confidence interval
Burdened by debt	0.555	0.01389	0.5278 to 0.5822
Would borrow less	0.544	0.01392	0.5167 to 0.5713
More hardship	0.343	0.01327	0.3170 to 0.3690
Loans worth it	0.589	0.01375	0.5620 to 0.6160
Career opportunities	0.589	0.01375	0.5620 to 0.6160
Personal growth	0.715	0.01262	0.6903 to 0.7397