1. Bernoulli Trials

2. Binomial distribution

3. Expected value,

4. Variance & sd (X)

5. Exercises

Homework: Ch. 4-1; 4-2
Binomial r.v.

Conditions:

1. The number of trials $n$ is fixed.

2. Bernoulli trials
   - Success $p = \mathbb{P}(\text{Success})$
   - Failure $q = 1 - p$

3. $p = \mathbb{P}(\text{Success}) = \text{const.}$

4. Independent trials

Let $X = \# \text{ of successes out of } n \text{ trials}$

$X = \text{Bin}(n, p)$

$f(k) = \mathbb{P}(X = k) = \binom{n}{k} p^k q^{n-k}, \quad k = 0, 1, \ldots, n$

$\mu = \mathbb{E}[X] = np \quad \text{and} \quad \text{Var}(X) = npq$
STAT 215

Binomial model

\[ \begin{array}{c|cc}
N_1 &=& 5 \\
N_2 &=& 5 \\
\hline
\begin{array}{c}
00000 \\
\ldots\ldots\ldots
\end{array} & \text{white} \\
\begin{array}{c}
\ldots\ldots\ldots
\end{array} & \text{black}
\end{array} \]

- \[ p = \text{the proportion of white} \]
- \[ 1-p = \text{the proportion of black} \]

**Drawing with replacement**

\[ \begin{array}{c}
\text{white} \\
\text{black}
\end{array} \]

\[ p = P(\text{white}) = \frac{1}{2} \]
\[ 1-p = P(\text{black}) = \frac{1}{2} \]

\[ n = 3 \quad \# \text{ of drawings} \]

\[ X = \# \text{ of white balls out of } n=3 \]

**Binomial distribution with** \( n=3 \) \& \( p=\frac{1}{2} \)

\[ P(X=k) = \binom{3}{k} \cdot p^k \cdot (1-p)^{3-k} \]

\[ P(X=0) = \binom{3}{0} \cdot \left(\frac{1}{2}\right)^0 \cdot \left(\frac{1}{2}\right)^3 = \frac{1}{8} \]

\[ p=\frac{1}{2} \]
\[
P(X=1) = \binom{3}{1} \left( \frac{1}{2} \right)^1 \left( \frac{1}{2} \right)^2 = 3 \times \frac{1}{8} = \frac{3}{8}
\]

\[
P(X=2) = \binom{3}{2} \left( \frac{1}{2} \right)^2 \left( \frac{1}{2} \right)^1 = 3 \times \frac{1}{8} = \frac{3}{8}
\]

\[
P(X=3) = \binom{3}{3} \left( \frac{1}{2} \right)^3 \left( \frac{1}{2} \right)^0 = 1 \times \frac{1}{8} = \frac{1}{8}
\]

\[
P(X=0) = \binom{3}{0} \left( \frac{1}{2} \right)^0 \left( \frac{1}{2} \right)^3 = \frac{1}{8}
\]

The sample space construction:

\[
S = \{ BBB, BBW, BWB, WBB, WWB, WWW \}
\]

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>1/8</td>
<td>3/8</td>
<td>3/8</td>
<td>1/8</td>
</tr>
</tbody>
</table>

The function \( f(x) \) is symmetric.

\[
\text{median} = \text{mean} = \frac{3 \times \frac{1}{2}}{3} = 1.5
\]
Binomial Distribution

Problem #1.

The probability of producing a defective item is 0.05.

(a) What is the probability that out of 20 items that are selected independently at least two are defective?

(b) How many defective items do you expect in a sample of 260 items?
Solution

Problem #1.

1st item

\[ \text{Def.} \quad p = 0.05 \]
\[ \text{Not Def.} \quad q = 0.95 \]

20th item

\[ \text{Def} \quad p = 0.05 \]
\[ \text{Not Def.} \quad q = 0.95 \]

Let

\[ X = \# \text{ of defectives out of } n = 20 \text{ items} \]

\[ X = \text{Bin}(n=20; p=0.05) \]

\[ f(k) = P(X = k) = \binom{20}{k} (0.05)^k (0.95)^{20-k} \]

for any \( k = 0, 1, 2, \ldots, 20 \)
a) \( P(\text{at least 2 are defective}) = \)

\[
P(X \geq 2) = \left[ \frac{P(X=0)}{P(X=1)} \right] \]

\[
= 1 - \overline{P(X \geq 2)} = 1 - P(X \leq 1) = 1 - F(1)
\]

\[
= 1 - P(X=0) - P(X=1) =
\]

\[
= 1 - \binom{20}{0} (0.05)^0 (0.95)^{20-0}
\]

\[
- \binom{20}{1} (0.05)^1 (0.95)^{20-1}
\]

\[
= 1 - (0.95)^{20} - 20 (0.05) (0.95)^{19}
\]

\[
= 1 - 0.358 - 0.377 = \boxed{0.265}
\]

i.e. \( P(X \geq 2) = \boxed{0.265} \)

Table C.2 → \( P(X \leq 1) = 0.7358 = F(1) \)

\( P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.7358 = \boxed{0.264} \)
(b) If \( n = 260 \) and \( p = 0.05 \)

let \( X^* = \# \) of defectives
out of \( n = 260 \)

\[ X^* = \text{Bin}(n = 260, p = 0.05) \]

What is expected number
of defectives?

\[ E(X^*) = n \cdot p \]

\[ = 260 \cdot (0.05) = \]

\[ = 13 \]

Table for Bin\( (n, p) \)