Normal Distribution:

Goals of the lab.
• To familiarize students with the Normal distribution.
• To have them gain an understanding of the z-score and its application to finding probabilities from the Normal distribution.

Background:
The Normal distribution is one of the most commonly used models for data. There are two basic reasons for this. First, it is a good approximation for a wide variety of data. Second, the Normal distribution can result from averaging or summing random variables, Normal or otherwise. The latter leads to things like the Central Limit Theorem that we will discuss in a few weeks. In addition it leads to almost all large sample inferential theory. Consequently, the Normal distribution is an important component of Statistics 211. For calculations the “Standard Normal” distribution is usually denoted by the letter Z, and has a mean of 0 and a standard deviation of 1.

There are some special issues that we deal with when using the Normal distribution because of our textbook. Table E of our textbook gives cumulative probabilities for positive and negative z-scores. As a consequence we need to have a method for dealing with probabilities greater than a certain z-score. Below is a template for using Table E in the textbook.

<table>
<thead>
<tr>
<th>What we want</th>
<th>Calculation we need to perform</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(Z&lt;z)</td>
<td>P(Z&lt;z)</td>
<td>P(Z&lt;1.42)</td>
</tr>
<tr>
<td>P(Z&gt;z)</td>
<td>P(Z&gt;z) = 1-P(Z&lt;z)</td>
<td>P(Z&gt;1.42) = 1-P(Z&lt;1.42)</td>
</tr>
</tbody>
</table>

Relevant Formulae:
If random variable X has mean $\mu$ and standard deviation $\sigma$, then we can compute

$$Z = \frac{X - \mu}{\sigma}$$

Example:

Suppose that H is a Normal random variable with mean 50 and standard deviation 3.

Find:

$$P(H<55) = P(Z<\frac{55-50}{3}) = P(Z<1.67) = 0.9525.$$

(directly from Table E)

$$P(H<57) = P(Z<\frac{57-50}{3}) = P(Z<2.33) = 0.9901.$$

(directly from Table E)

$$P(H>54.8) = P(Z>\frac{54.8-50}{3}) = P(Z>1.60) = 1 - P(Z<1.60) = 1 - 0.9452 = 0.0548.$$

$$P(H<49.2) = P(Z<\frac{49.2-50}{3}) = P(Z<-0.27) = 0.3936.$$

(directly from Table E)

$$P(H>48.6) = P(Z>\frac{48.6-50}{3}) = P(Z>-0.47) = 1 - P(Z<-0.47) = 1 - 0.3192 = 0.6808.$$