**Probability Definitions:**

*Goals of the lab.*
- To introduce some concepts of probability
- To familiarize students with basic concepts of probability including sampling with and without replacement.

*Background:*

Basic Definitions:

A **trial** is an action that results in one of several outcomes.

An **experiment** is a single trial or a series of trials.

The **sample space** of an experiment is the set of all possible outcomes for the experiment.

An **event** is a subset of the sample space.

There are two basic definitions of probability that will be used in this course:

1) Suppose that an experiment can result in one of \( m \) equally likely outcomes. Suppose also that event \( E \) consists of \( r \) of these \( m \) possible outcomes. Then the **theoretical probability** of event \( E \) is

\[
P(E) = \frac{r}{m} = \frac{\text{number of distinct outcomes for which event } E \text{ occurs}}{\text{total number of possible outcomes}}
\]

2) The **relative frequency definition of probability** (or **experimental probability** or **empirical probability**) of an event is the proportion of times that the event will occur if the experiment is repeated over and over for a very long time. Suppose that an experiment consists of \( n \) trials (where \( n \) is very large) and \( k \) of these trials results in event \( E \). Then the **experimental probability** of event \( E \) is

\[
P(E) = \frac{k}{n} = \frac{\text{number of trials in which event } E \text{ occurred}}{\text{total number of trials conducted}}
\]

The "Law of Large Numbers" states that the experimental probability of event \( E \) approaches the theoretical probability of event \( E \) as \( n \), the number of trials conducted, approaches infinity.

The other topic of interest is sampling with and without replacement. These concepts are involved when you are selecting more than one object. Sampling with replacement implies that once you take the first (or later) object from the list, you return it to the list for possible selection again. Sampling without replacement implies that the selected object (or objects) is not returned to the list for possible subsequent selection. This is important when computing conditional probabilities.

*Relevant Formulae:*

See above.
Examples:

Suppose you have a box of 50 “8-track” tapes. There are 5 red Jazz tapes, 10 red Rock tapes and 5 red Swing tapes. 10 blue Jazz tapes, 10 blue Rock tapes and 10 blue Swing tapes. Select one tape from the box.

Let J be the event that the selected tape is a Jazz tape. Let B be the event that the selected tape is a blue tape. Let R be the event that the selected tape is a Rock tape.

\[ P(J) = \frac{15}{50} = 0.30 \]
\[ P(B) = \frac{30}{50} = 0.60 \]
\[ P(R) = \frac{20}{50} = 0.40 \]

A fair coin is tossed several times and the number of times that “Heads” appears is recorded in the table below:

<table>
<thead>
<tr>
<th># tosses</th>
<th># Heads</th>
<th>( \hat{p}(\text{Heads}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>46</td>
<td>0.460</td>
</tr>
<tr>
<td>500</td>
<td>261</td>
<td>0.522</td>
</tr>
<tr>
<td>1000</td>
<td>518</td>
<td>0.518</td>
</tr>
<tr>
<td>5000</td>
<td>2488</td>
<td>0.4976</td>
</tr>
<tr>
<td>10000</td>
<td>5012</td>
<td>0.5012</td>
</tr>
</tbody>
</table>

Note that experimental probability \( \hat{p}(\text{Heads}) \) tends to approach the theoretical probability \( P(\text{Heads}) \) as \( n \), the number of tosses, increases.