**Poisson Random Variable**

The second discrete RV that we will cover in this chapter is the Poisson RV. Roughly speaking the Poisson RV is about counts. How many things happen in an interval of time or space is what this random variable is about. One of the most common uses for this variable is in predicting how many failures will occur for a particular mechanical system, e.g. how many times a copier breaks in a year, or how many imperfections there are on a plate of glass. The Poisson random variable is named after a French mathematician,

**Definition:** A Poisson RV Y is a discrete RV that has the following characteristics:

1. Events occur randomly in a unit of time, area or space.
2. The number of events has no fixed upper limit.
3. The mean number of events is constant per unit of time, area or space

Y is then the number of events that occur in the unit of time, area or space.

We describe the distribution for a Poisson RV with the following formula

If we know $\lambda$,

$$P(Y=y) = \frac{e^{-\lambda} \lambda^y}{y!}$$

for $y = 0, 1, 2, 3, \ldots$

$\lambda$ is used in this formula and represents the average rate of events. That is, $\lambda$ is the average number of events that occur in the unit of time or area or space. Also in the above formula, the $e$ represents a number known as Euler’s (pronounce Oiler’s) constant.

The **mean** of a Poisson RV Y is $\mu_y = \lambda$

The **standard deviation** of a Poisson RV X is $\sigma_y = \sqrt{\lambda}$

**Example:**

Farmers are often concerned with the pests on their crops. The Oakridge beetle is an especially damaging pest for turnip crops. During the harvest season in Ohio, the average number of Oakridge beetles on 4 acres of turnip is 3.2. What is the chance that during the next harvest season, there will be no beetles on 4 acres found in Melessa, Ohio.

Let $Y$ be the number of beetles found on the 4 acre turnip plot in Melessa, Ohio. Are the conditions met?

1. Do the events randomly occur in time and space? Yes, we can believe that the beetles random arrive on the 4 acre plot.
2. Does the number of events have no fixed upper limit? Yes, it is conceivable that there could be lots and lots of beetles on this 4 acre plot.

So we need to find the $P(Y=0)$?

Since the average number of beetles is 3.2, then $\lambda = 3.2$.

So,

$$P(Y=0) = \frac{e^{-\lambda} \lambda^y}{Y!} = \frac{e^{-3.2} 3.2^0}{0!} = (0.0408)*(1.0)/1.0 = 0.0408.$$  

Recall that $0! = 1$.

Thus, the probability of seeing no beetles on the 4 acre plot is 0.0408.
Example:
Stanton Airlines is the major airlines operating in Australia. They run flights everyday both within Australia and to other parts of the world. Officials there want to determine the number of maintenance delays they will have to deal with during the upcoming holiday week. They know that on average they have 3.4 maintenance delays per week. Fearing a possible strike, they want to calculate what is the chance that they experience 7 maintenance delays next week.

First, we must ask are the conditions met for using a Poisson distribution?
1. Are maintenance delays random events in a unit of time or space? Yes, we are only looking a week of time so that we are dealing with a fixed unit of time and we can reasonably assume that maintenance delays are random events.
2. Is there no maximum on the number of delays for maintenance? That may be a stretch, but it is possible that they could have lots and lots of delays, so this assumption will not be too badly violated.

Since the conditions are met we can use the Poisson RV to calculate the probability of interest.

Let $D$ be the number of delays they experience next week. Since the average number of delays is 3.4, $\lambda = 3.4$. Then,

$$P(D=7) = \frac{e^{-\lambda} \lambda^d}{d!} = \frac{e^{-3.4} \cdot 3.4^7}{7!} = (0.0334) \cdot (5,252.335)/(5040) = 0.0348.$$ 

Thus there is only a 3.5% chance that they will experience 7 maintenance delays next week.

**TIP:** It is not always clear when a condition for the use a particular random variable might be violated. Without a great deal of experience and mathematical sophistication it is hard to know the consequences of violating a particular condition. A good rule of thumb is to report any possible violation.

**TIP:** When reporting probabilities we will use three or four decimal places since that is what our textbook uses.
For the cumulative Poisson probabilities, we will use Table 2 in the textbook. With the binomial tables we needed the to know the values of n and p, as well as r, to use the binomial tables. Since the Poisson distribution is determined by \( \lambda \), then we simply need to identify \( \lambda \) and r. This makes the Poisson table slightly easier to use. To use Table 2, we need to find the appropriate value for \( \lambda \) in the row labels. Having found the row for \( \lambda \), we find the column for r and where they intersect is the probability that we’re interested in.

**Example:**
Find \( P(X \leq 3) \) with \( \lambda = 2.2 \).
On page 510 has \( \lambda = 2.2 \) near the bottom of the page. We then find the column for \( x = 3 \). Where the row for \( \lambda = 2.2 \) and the column for \( x = 3 \) intersect, we find 0.819 which is the probability we need.
So \( P(X \leq 3) = 0.819 \).

**Example:**
Find \( P(X \leq 6) \) with \( \lambda = 4.6 \).
The row for \( \lambda = 4.6 \) is found on page 511 near the top of the page. We then find the column for 4.6. Then we find the column where \( x = 6 \). Finding the element that is in appropriate row and column, it is 0.8180.
So \( P(X \leq 6) = 0.818 \).

Finally we noted that both the Binomial and Poisson Tables in our textbook are incomplete. They do not contain all possible values for n and p or for \( \lambda \).

**TIP:** One quick way to determine whether you are dealing with a binomial or a Poisson random variable is to ask whether or not there is a fixed maximum number. If there is, then the binomial may be appropriate; if there is not, then the Poisson may be appropriate. As always you still need to check the appropriate conditions for each RV.

**TIP:** For the binomial (or the Poisson), if the values that you need for n and p (or m) are not in the tables, then it’s back to using the formula to calculate the appropriate probabilities.